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(2) Here again are given two circles tangent internally, but no definite ratio is specified between the lengths of the radii. Take the point of tangency as the origin and the common tangent to the two circles as the x-axis, the y-axis being perpendicular to the x-axis. Draw the secant line as before and also the lines through R and S parallel to the x- and y-axes, respectively, and meeting at P. Required the locus of P as the secant line revolves. See Fig. 2.

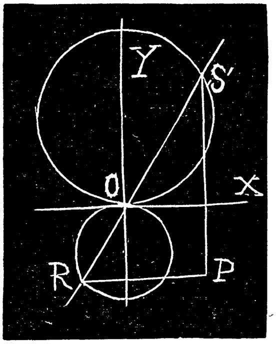

Fig. 3.

(3) This construction is similar to that of (2) except that the two circles are tangent externally, and the common tangent to the circles at the point of tangency is taken as the x-axis. See Fig. 3.

(4) Here the center of one circle lies on the circumference of the other circle. This center is taken as the origin, and the line joining it to the center of the other circle is taken as the y-axis. The secant and the other lines are drawn as in the other constructions, and the locus of P is required. See Fig. 4.

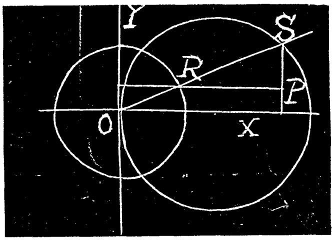

Fig. 4.

All four of these constructions can be shown by very elementary considerations to lead to conics, two of them giving ellipses, and the other two giving parabolas. Constructions similar to these can be made to yield an indefinite number of curves by simply replacing one or both of the circles by ellipses, parabolas, hyperbolas, or other curves. Such considerations, however, would lead us beyond the scope of an elementary course.

DEPARTMENTS.

SOLUTIONS OF PROBLEMS.

ALGEBRA.

368. Proposed by E. B. ESCOTT, University of Michigan, Ann Arbor, Michigan.

Solve the functional equation, $\dfrac{f(-x)}{f(x)}=r^{2x}$.

I. Solution by the PROPOSER.

Let $f(x)=A(x^2)+xB(x^2)$. Then the equation becomes

$$\frac{A(x^2)-xB(x^2)}{A(x^2)+xB(x^2)}=r^{2x};\ \text{whence}\ \frac{A(x^2)}{x\,B(x^2)}=\frac{1+r^{2x}}{1-r^{2x}}=-\frac{r^x+r^{-x}}{r^x-r^{-x}};$$

$$\text{or, }\frac{A(x^2)}{B(x^2)}=-\frac{r^x+r^{-x}}{(1/x)\ (r^x-r^{-x})}.$$

$\therefore A(x^2)=(r^x+r^{-x}).\phi(x^2)$, $B(x^2)=-\dfrac{1}{x}(r^x-r^{-x}).\phi(x^2)$, where $\phi(x^2)$ is any function of x^2.

$\therefore f(x)=\dfrac{2}{r^x}.\phi(x^2)$.

II. **Solution by S. LEFSCHETZ, Ph. D., The University of Nebraska.**

The given functional equation can be written:

$$f(-x)r^{-x}=f(x)r^x.$$

Hence $f(x)r^x=\phi(x)$, where ϕ is any even function of x.

$\therefore f(x)=\dfrac{\phi\ (x)}{r^x}$.

Ex. $\phi(x)=\sum\limits_{0}^{\infty} mA_m\cos mx$; $\phi(x)=\psi(x^2)$, etc.

Also solved by A. H. Holmes and J. Scheffer.

369. **Proposed by WILLIAM HOOVER, Ph. D., Athens, Ohio.**

If $f(m)=(1+x)^m$, and $f(n)=(1+x)^n$, why not obviously $f(m).f(n)=(1+x)^{m+n}=f(m+n)$?

Solution by J. SCHEFFER, A. M., Hagerstown, Maryland.

The proof given of $f(m)f(n)=f(m+n)$ is certainly incontestable; but it may also be proved directly thus: Differentiating with reference to m and n separately, we have

$$f(n)\,f'\,(m)=f'\,(m+n),\quad f'\,(n)\,f(m)=f'\,(m+n).$$

$\therefore f(n)\ f'\ (m)=f'\ (n)\ f(m)$; $\therefore \dfrac{f'\ (m)}{f(m)}=$a constant$=a$ (say).

$\therefore f(m)=a^m$.

Solutions of 367 were received from A. M. Harding, Elmer Schuyler, S. Lefschetz, and the Proposer.